Assimilation of Oceanic Observations With a Reduced Order Square-Root Smoother

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myOcean WP3

MY OCEAN

Marine Core Service

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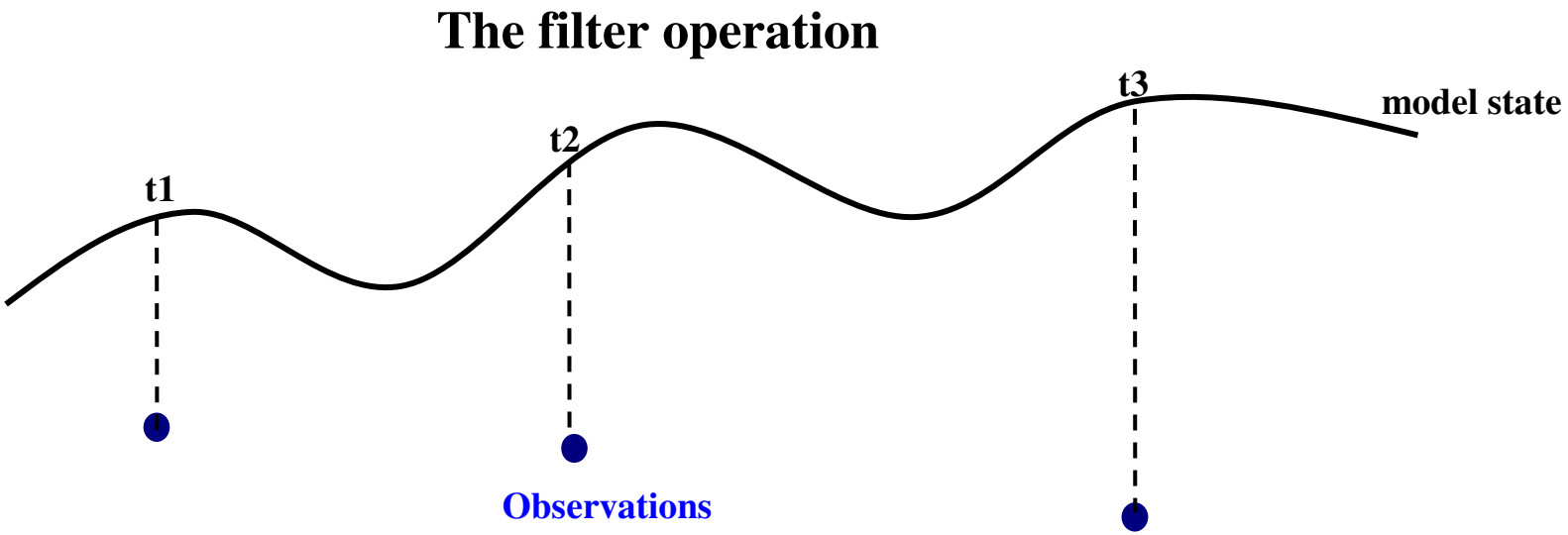
1. Introduction

- **WP3, Section 3.1: Research and Developments**
- **Task 3.1.2: New design and implementation of statistical data assimilation schemes**
- **Sub-task a: 4-dimensional extensions of statistical assimilation algorithms**
- **Objective: Design assimilation algorithm for the specific purpose of reanalyses**

1. Introduction

- **Present reanalyses are produced using a filter**



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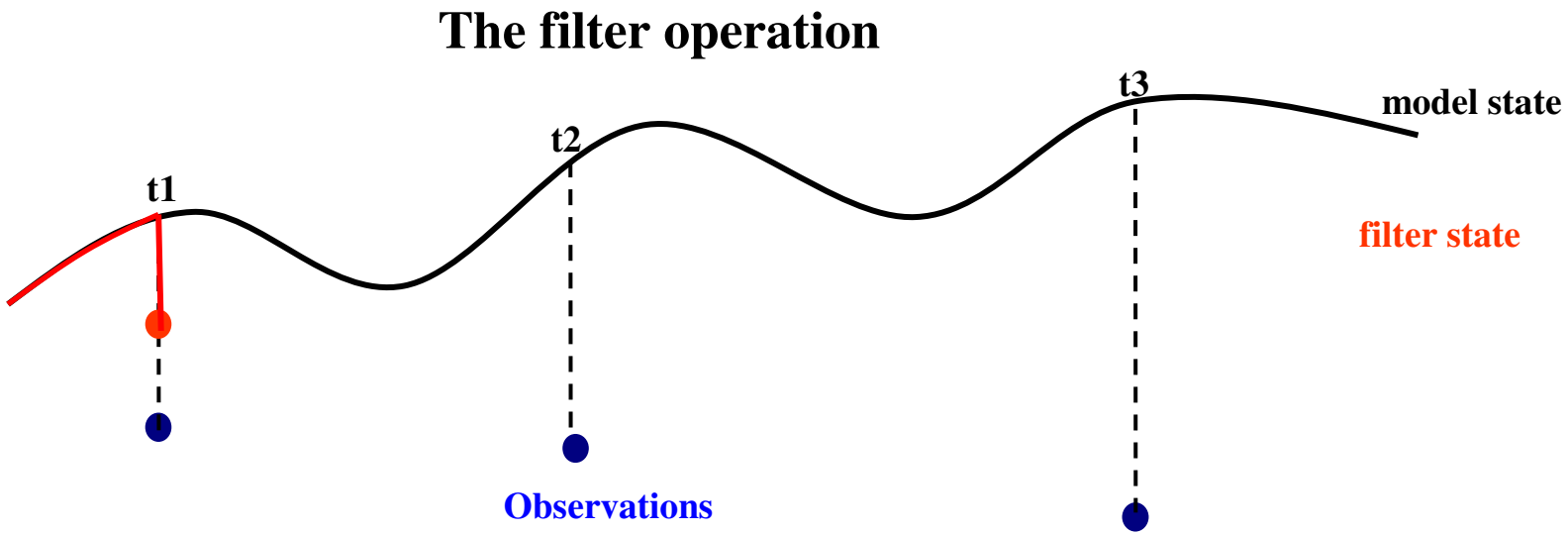

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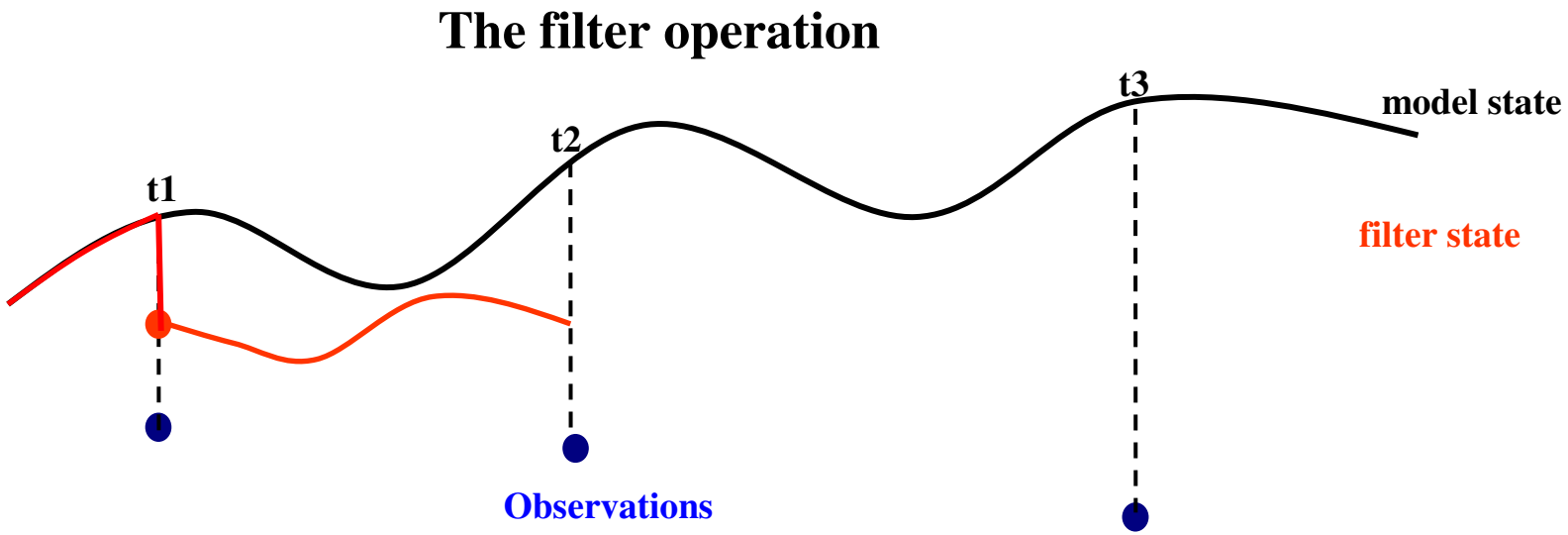

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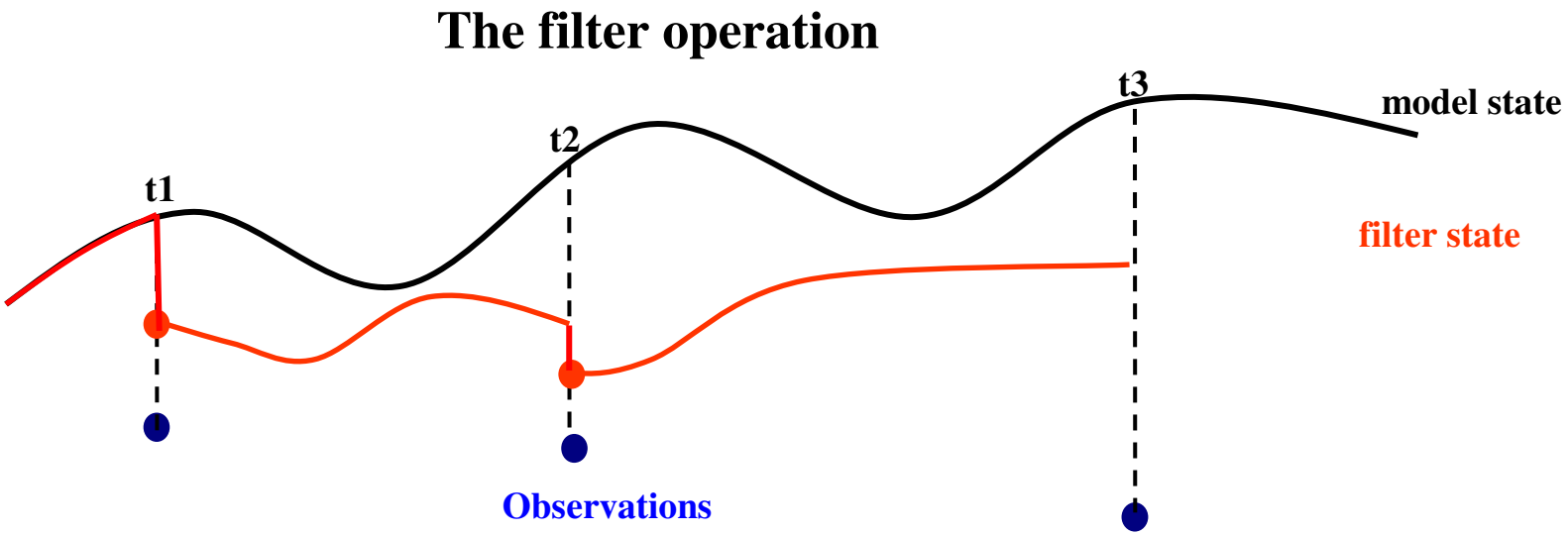

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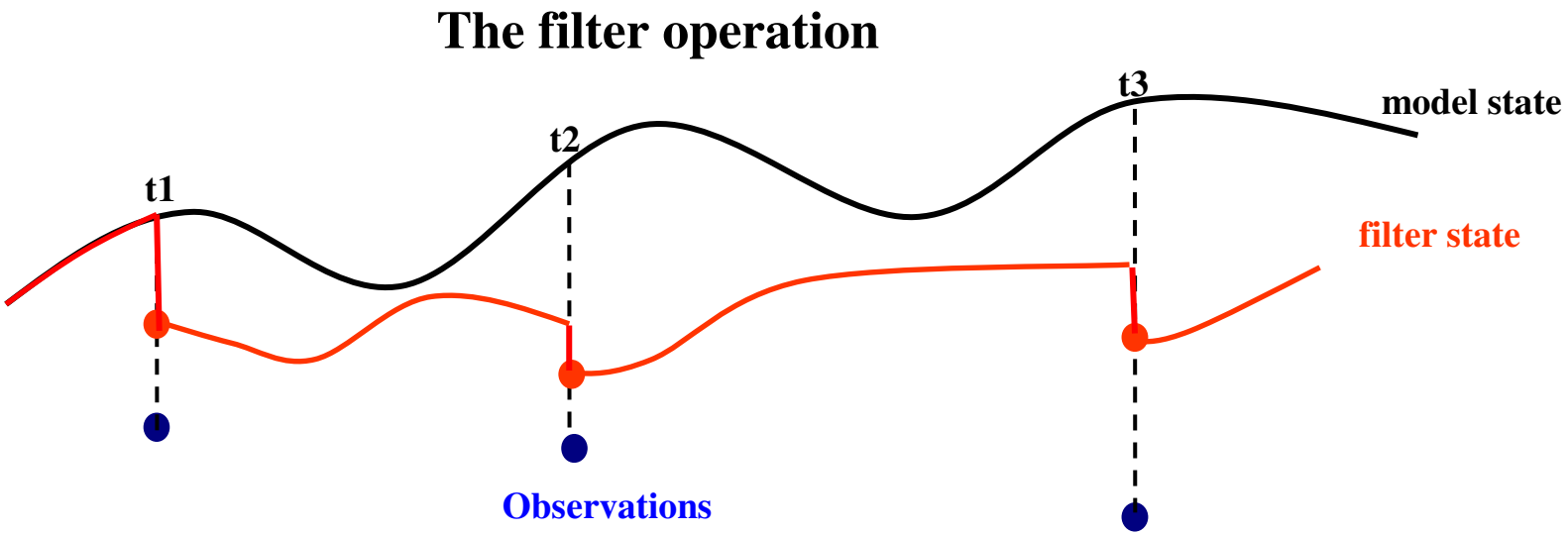

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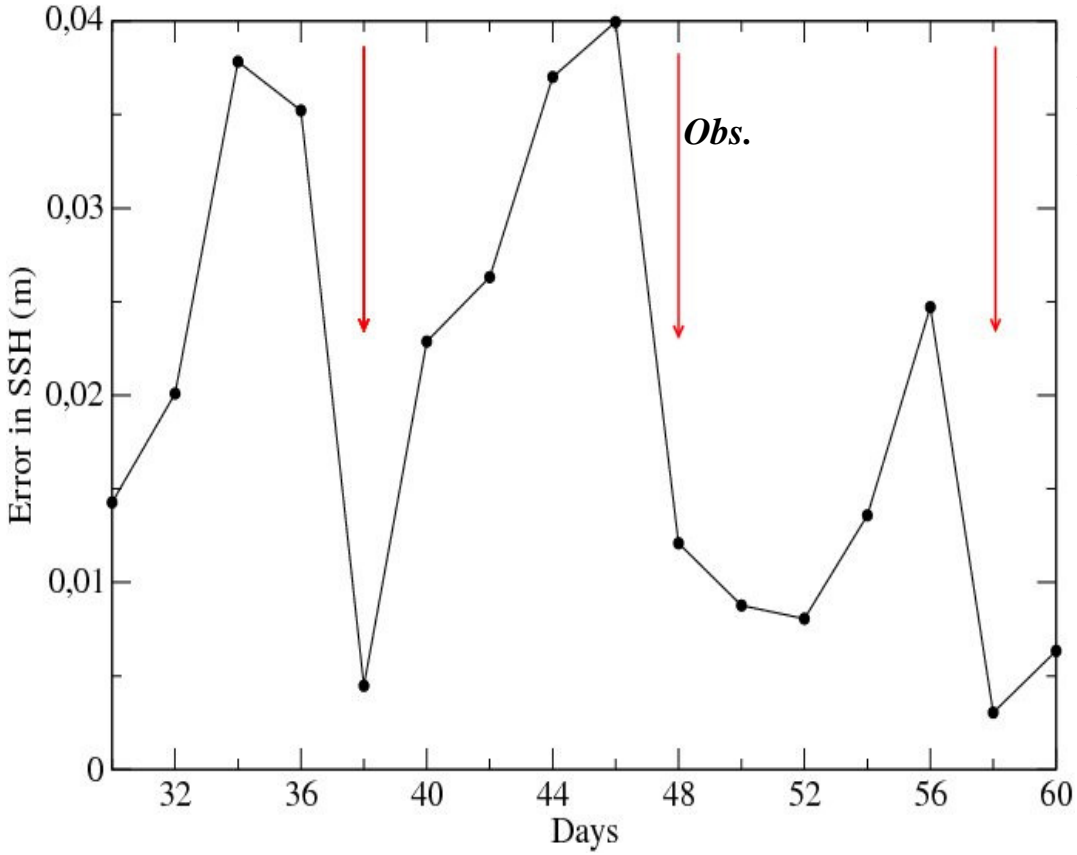
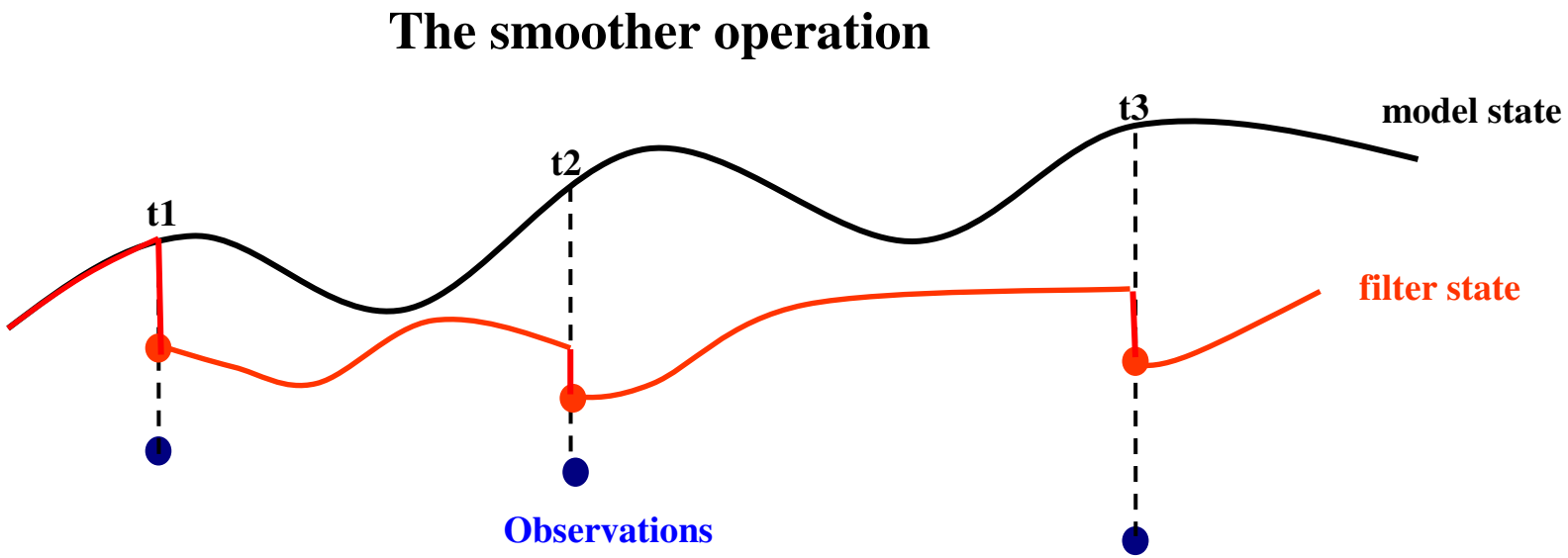


Evolution of error in SSH at one grid point, in a Kalman Filter experiment with TOPEX /POSEIDON observations every 2 days.

- **The error decreases when close observations are available**
- **The filter provides a sequence of estimations of heterogeneous quality**
- **A Smoother may solve this**

1. Introduction

- **The smoother uses observations to correct retrospectively the previous analysis states.**



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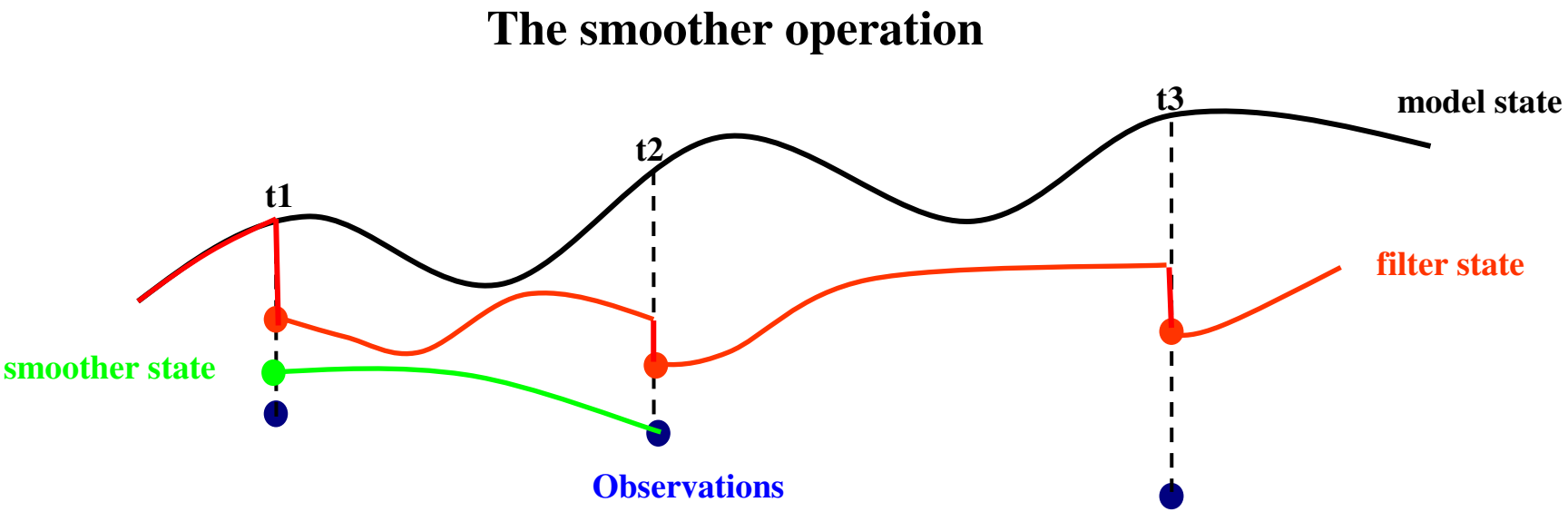

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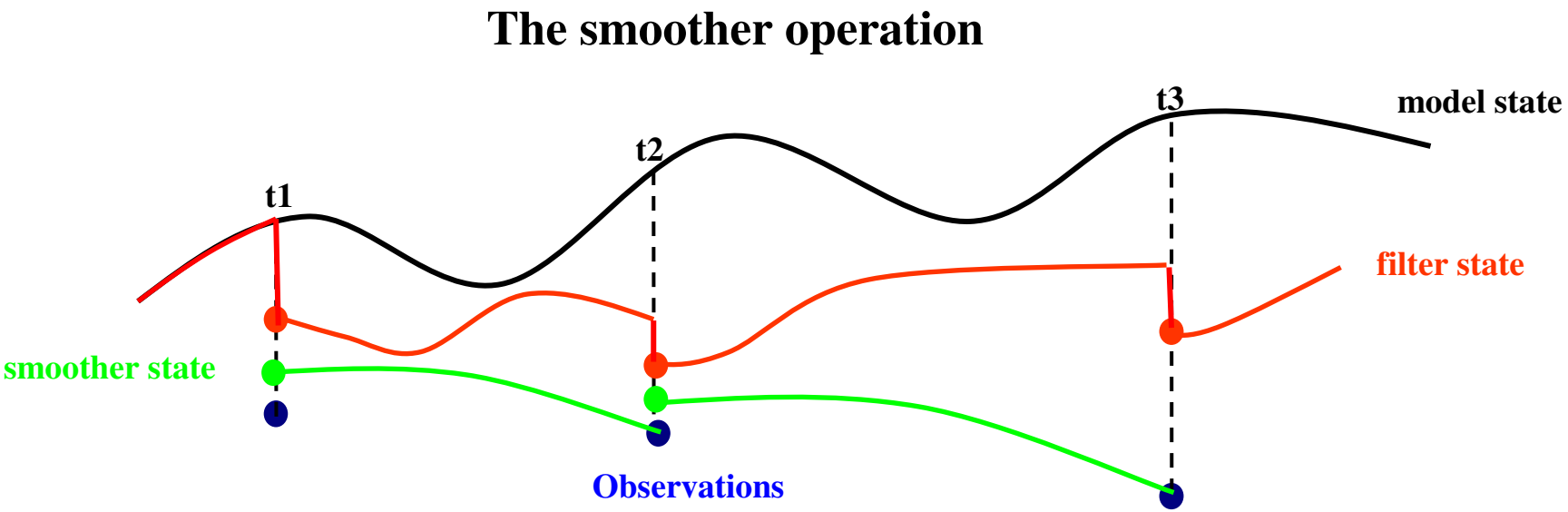
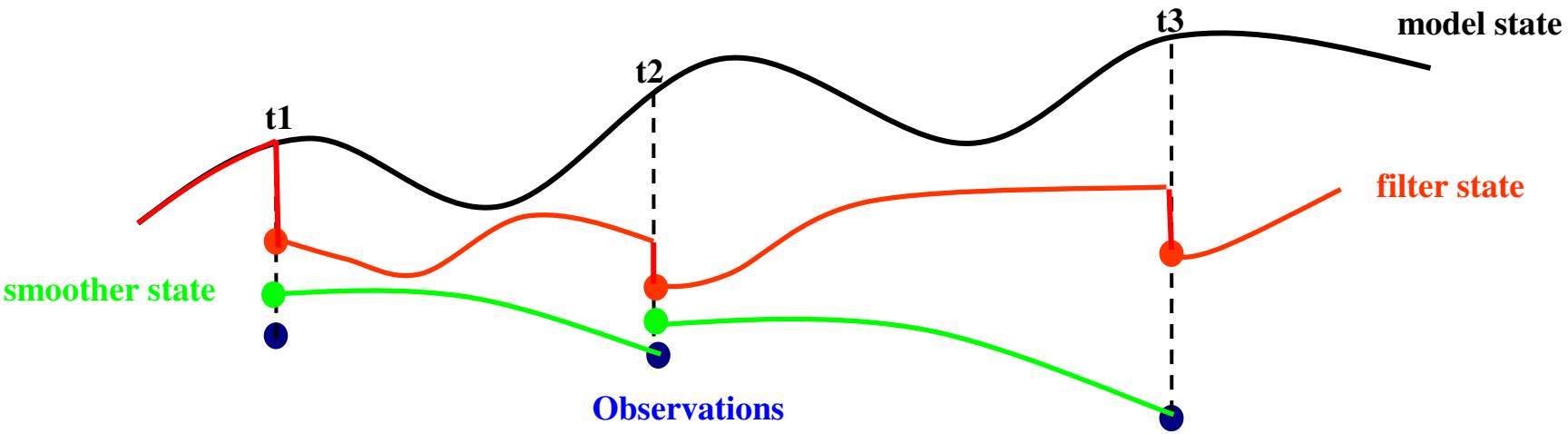


Outline

- Introduction
- **Completed developments: The SEEK smoother**
- **Undergoing developments: adaptive model error**
- **Conclusions and perspectives**

2. The SEEK smoother

- **Observations are used to correct retrospectively the previous analysis states.**



2. The SEEK smoother

- **Equations:** Cosme et al. 2010

  - **Forecast step:**
    - $X^f_{k|k-1} = M_{k-1,k}\, X^a_{k-1|k-1}$ ←------- state propagation
    - $S^f_{k|k-1} = M_{k-1,k}\, S^a_{k-1|k-1}$ ←------- error propagation

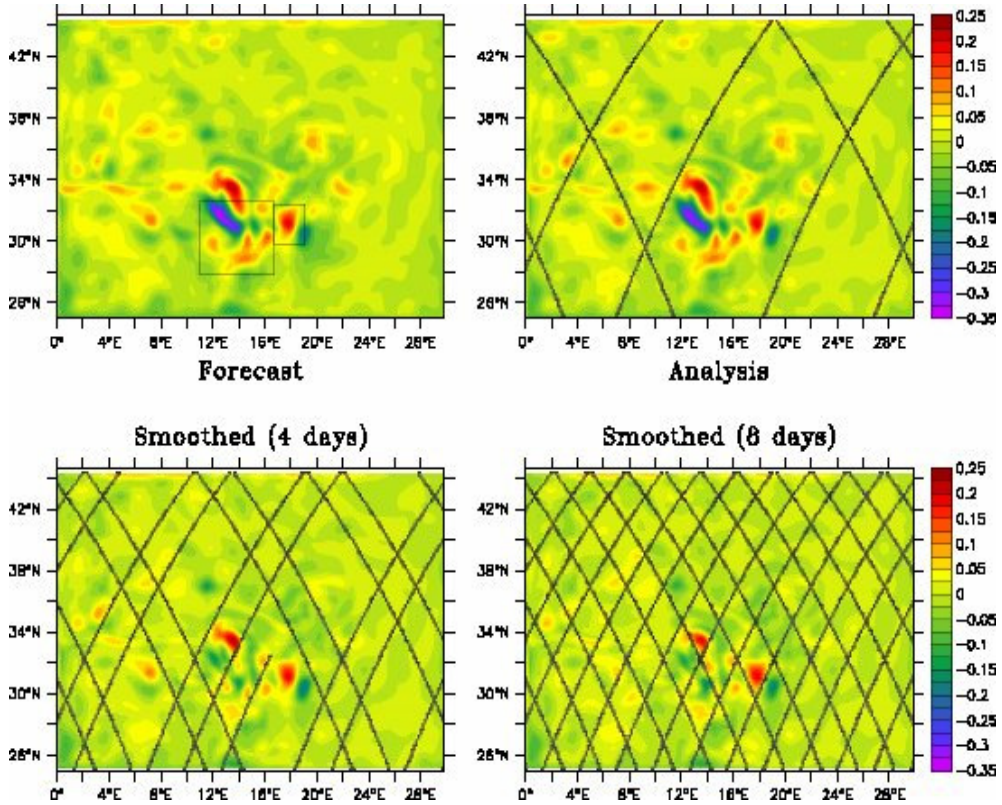
  - **Filter analysis step:**
    - $X^a_{k|k} = X^f_{k|k-1} + K_{k|k} d_k \quad , \quad K_{k|k} = S^f_{k|k-1} Z_k$ ←------- filter analysis
    - $S^a_{k|k} = S^f_{k|k-1} [I + \Gamma_k]^{-½}$ ←---- filter analysis (cov.)

  - **Smoother analysis step:**
    - $X^a_{i|k} = X^a_{i|k-1} + K_{i|k} d_k \quad , \quad K_{i|k} = S^a_{i|k-1} Z_k$ ←------- smoother analysis
    - $S^a_{i|k} = S^a_{i|k-1} [I + \Gamma_k]^{-½}$ ←--- smoother analysis (cov.)

    Less cost in addition to the filter

2. The SEEK smoother

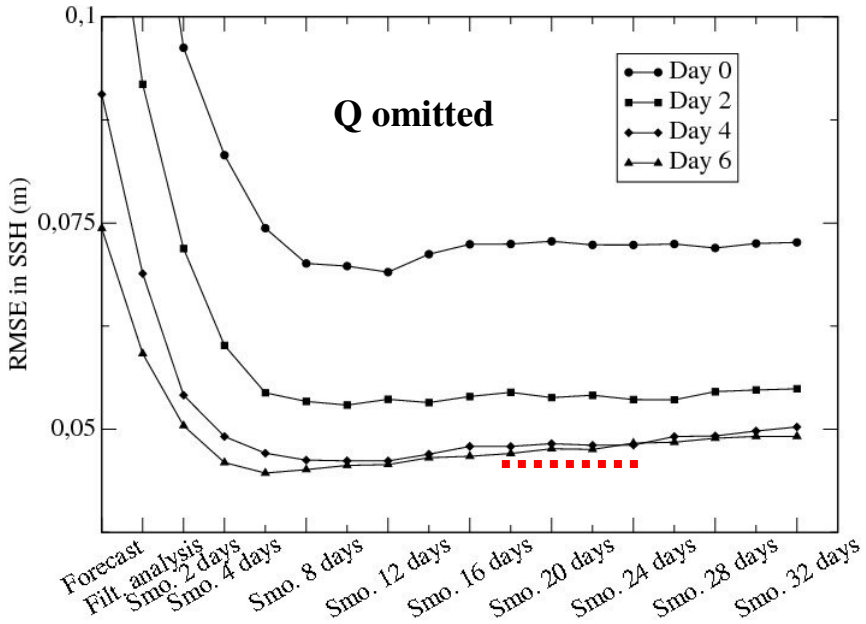
- **An idealised double-gyre configuration of NEMO: error on SSH**

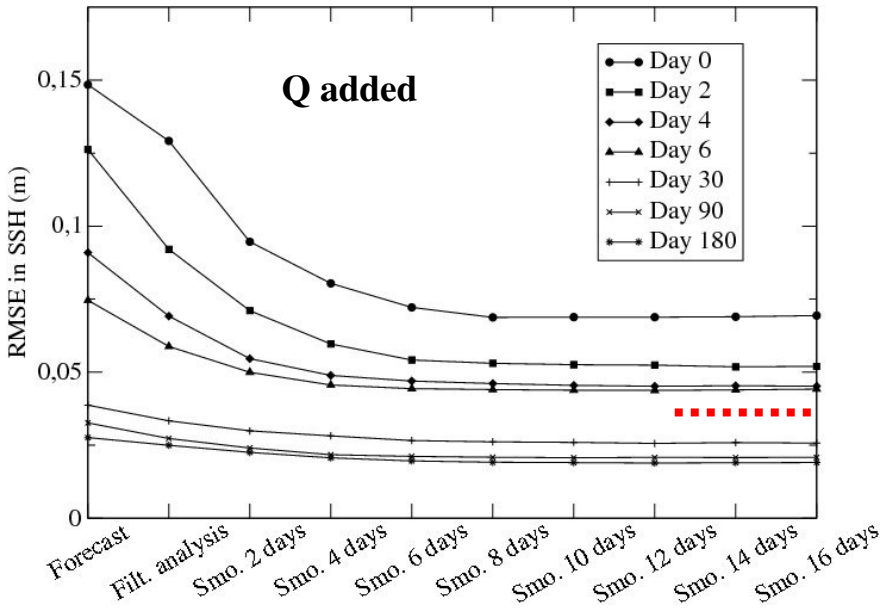


- **The smoother reduces significant errors still present after the analysis**

2. The SEEK smoother

- **Error in SSH plotted against the assimilation step, for different days**





- **The model error should make temporal correlations fade and limit the impact of observations distant in time**
- **An accurate model error parameterisation is essential for the smoother**

2. The SEEK smoother

- Implement with a realistic NEMO configuration
- Improve the model error parameterisation

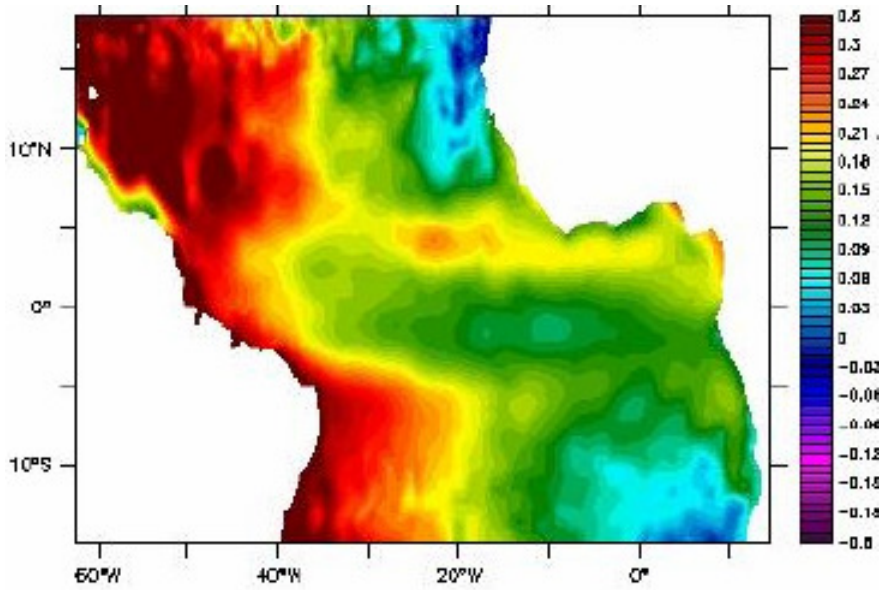
2. The SEEK smoother

- **Regional configuration of the Tropical ATLantic ocean at a resolution of ¼ degree** (*PhD thesis of N. Freychet*)

➢ **TATL4 configuration of the NEMO ocean model** (Ubelmann, 2009)

➢ **Good benchmark to test the smoother:**

- **Presence of Tropical Instability Waves (*TIWs*)**
  - ✓ **Fast oscillation traveling from East to West**
  - ✓ **Difficult to tract with "forward only" data assimilation method**
- **The North Brazil coastal region exhibits dynamics of the « mid-latitude » type (rings).**



a/ Observed SSH

2. The SEEK smoother

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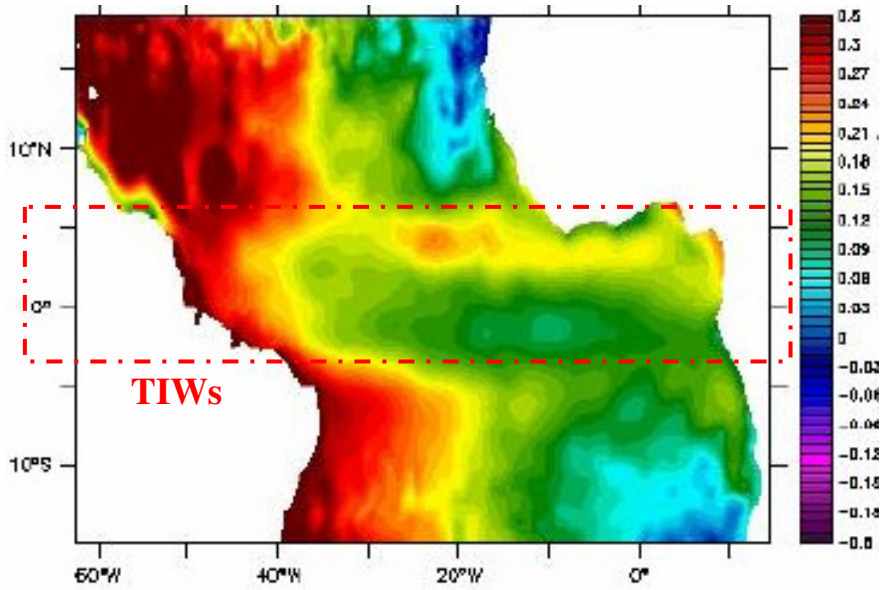

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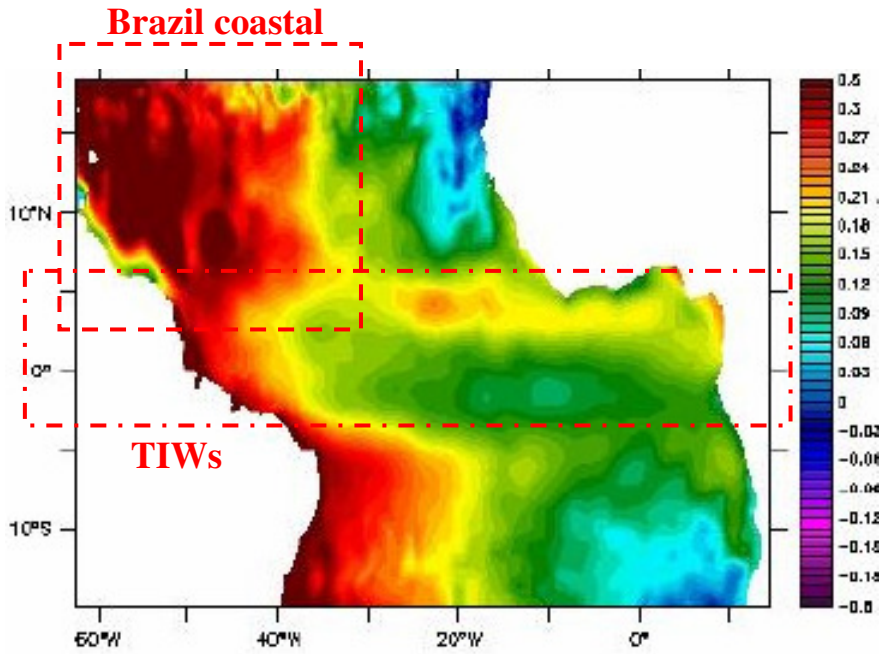
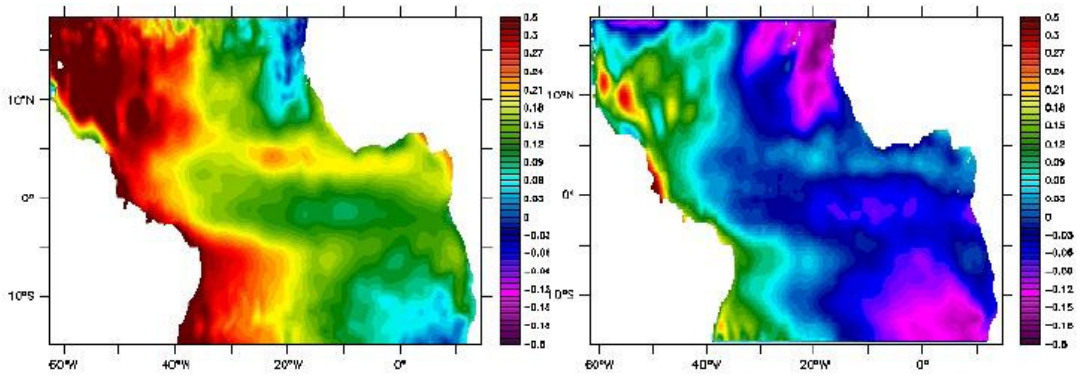


a/ Observed SSH

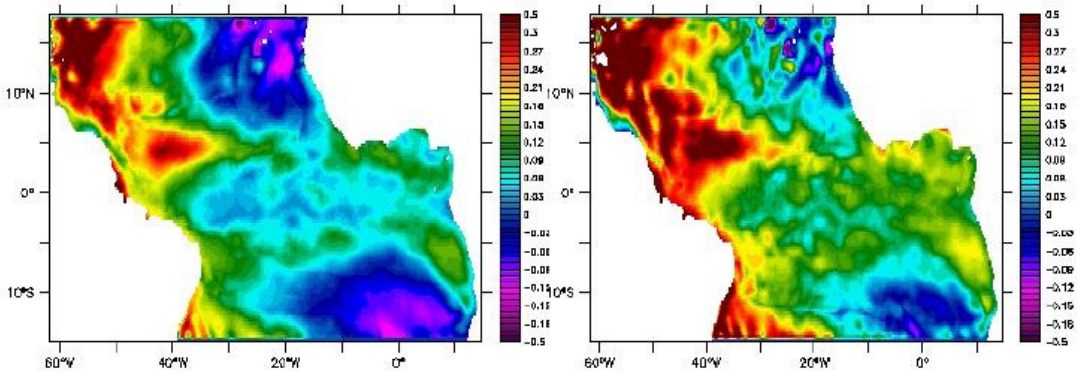
2. The SEEK smoother

- **SSH fields from the true state, the filter forecast, the filter analysis, and the 8-day smoother analysis.**



a/ Observed SSH

b/ Forecast SSH



c/ Analysed SSH

d/ Smoothed SSH

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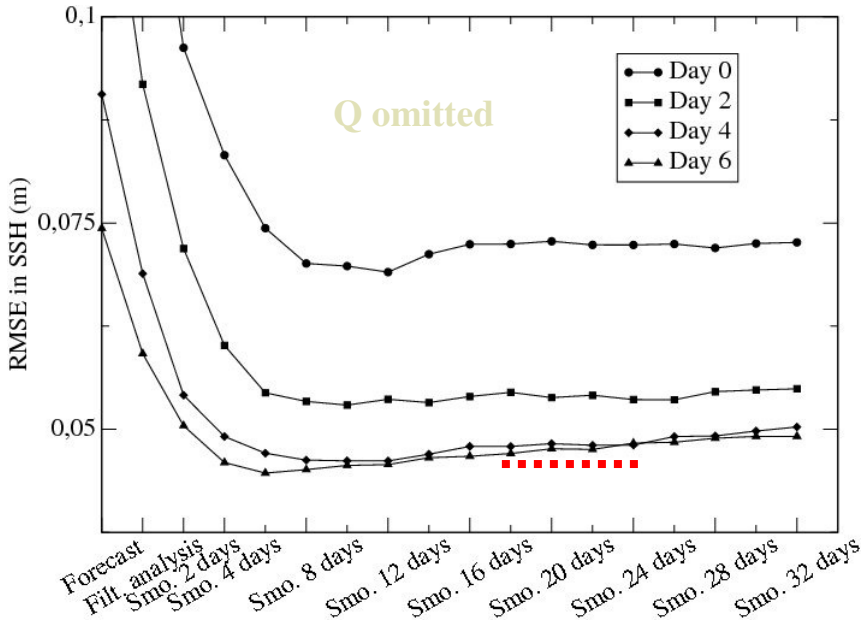
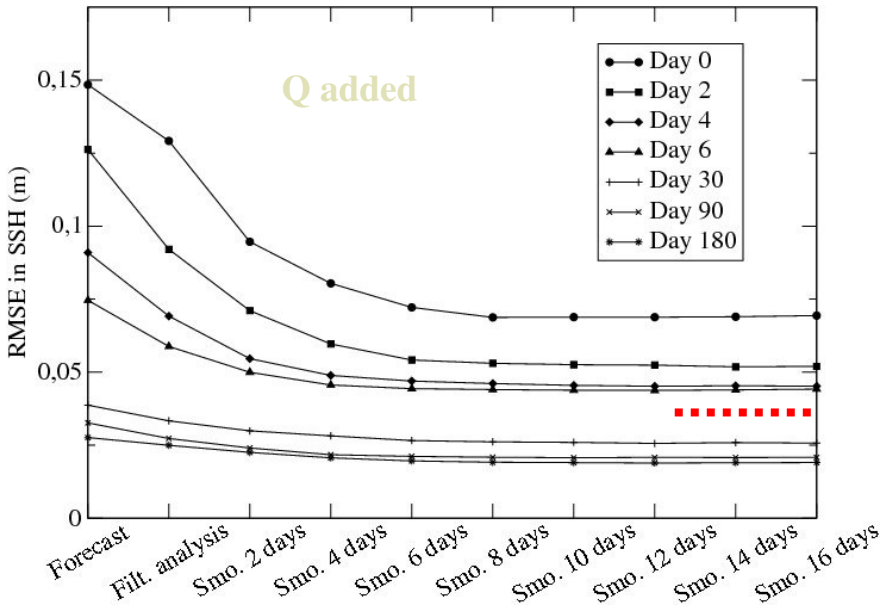
3. Undergoing developments

- Error in SSH plotted against the assimilation step, for different days





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3. The adaptive model error

- **How to model model error ?**
  - **Dee, 1995**
  - **Mitchell and Houtekamer, 2000**
  - **Anderson, 2007**
  - **Li and Kalnay, 2007**
  - **Danforth and Kalnay, 2008**
  - **Houtekamer and Mitchell, 2009**

Our approach follows Dee (1995)

3. The adaptive model error

- **The adaptive scheme of Dee (1995):**
  - **Background theory: The innovation $d = y - H(X^f)$ is the only objective source of information on errors**
  - **The innovation verifies: $< dd^T > = HP^fH^T + R$**
  - **Idea: write $P^f = P^f(\alpha)$ and find α so that theory is verified at best**
  - **In practise: search for α that minimizes the cost function :**

$$J(\alpha) = \log [\det C(\alpha)] + d^T C(\alpha)^{-1} d \qquad \text{with} \quad C(\alpha) = HP^f(\alpha)H^T + R(\beta)$$

3. The adaptive model error

- **Our adaptive scheme (Brankart et al, 2010) is an extension of the scheme of Dee (1995) with:**
  - ***R = R(β)*** **thus** ***C = C(α, β)*** **and** ***J = J(α, β)***
  - **The cost function is reformulated to take advantage of the reduced order approach of the SEEK filter:**

❑ **Cost function in innovation space : Dimension n x n**

$$J(\alpha,\beta) = -f \ln\left[p^{0}(\alpha,\beta)\right] + \frac{1}{2}\sum f\left[d^{T}C^{-1}(\alpha,\beta)d + \ln\left|C(\alpha,\beta)\right|\right]$$

❑ **Cost function in reduction space : Dimension r x r**

$$J^{A} = d^{T}R^{-1}(\beta)d - \delta^{T}(\alpha,\beta)\left[I+\Gamma(\alpha,\beta)\right]^{-1}\delta(\alpha,\beta)$$

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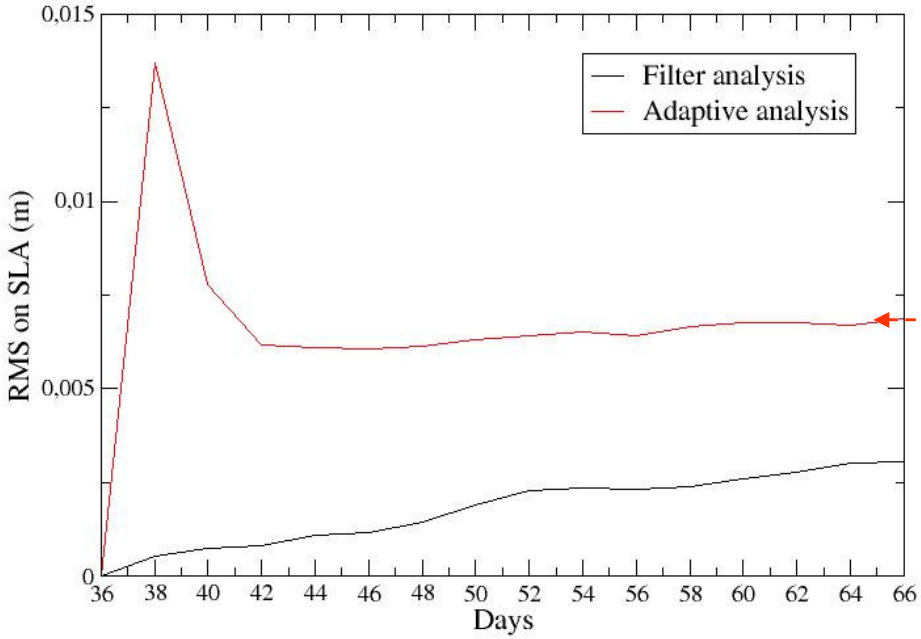
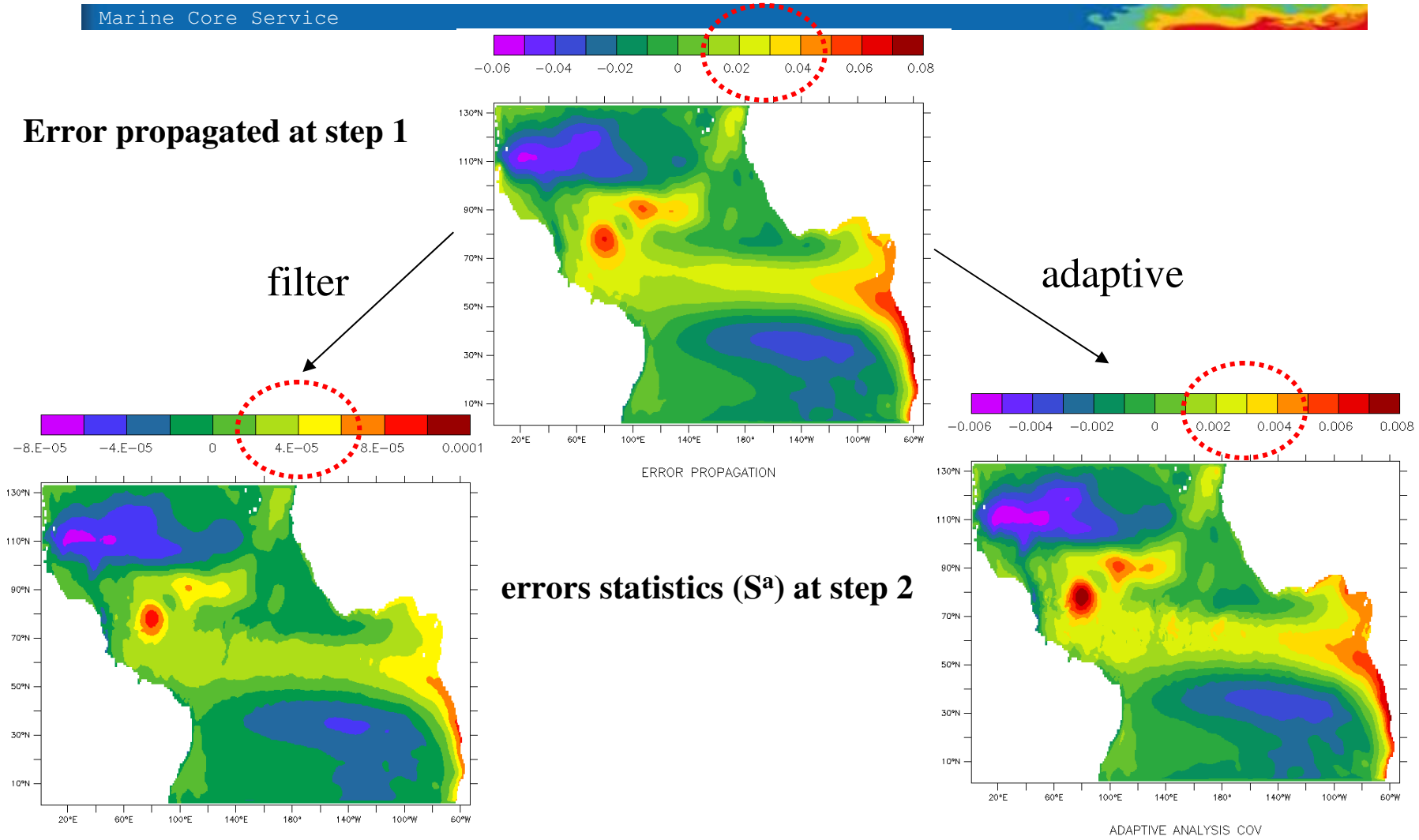
3. The adaptive model error

- With NEMO ocean model in global analysis on TATL4
- RMS in SSH of filter analysis compared to adaptive analysis



The adaptive error is logical and realistic

3. The adaptive model error



The adaptive could respond for the low parameterisation of error statistics

4. Conclusions

- **Summary:**
  - **Smoothers are expected to perform better than filters for reanalysis purposes;**
  - **The SEEK smoother has been developed and successfully tested with an idealised configuration of NEMO; Its numerical cost is very low;**
  - **The implementation with a realistic configuration is under way;**
  - **An accurate parameterisation of the model error is essential for the smoother.**
  - **The development of an accurate model error parameterisation is under way. First results are encouraging.**

4. Conclusions

- **Perspectives:**
  - **Adapt the adaptive scheme to local analysis;**
  - **Evaluate the benefit of the adaptive scheme with the smoother.**
  - **Implement the smoother in reanalyses-making systems, and compare with presently used algorithms.**

- **Thank you for your attention**