Particle Kalman Filtering for Nonlinear Data Assimilation

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Optimal nonlinear filtering consists of determining the conditional probability distribution function (*pdf*) of the state given previous measurements. Once the state *pdf* is known, one can determine different estimates of the system state, as the minimum variance estimate. Particle filters (PF) are discrete nonlinear filters that use point-mass representation (Dirac mixture) of the state *pdf*. In practice, these filters suffer from the degeneracy of its particles that causes very often the divergence of the filter. Another discrete solution of the optimal nonlinear filters is based on Gaussian sum representation of the state *pdf*. This results in a hybrid particle-Kalman filter in which the standard weight-type PF correction is complemented by a KF-type correction for each particle using the associated covariance matrix in the Gaussian sum. We refer to this filter as the particle Kalman filter (PKF). The solution of the nonlinear filtering problem is then obtained as the weighted average of an ensemble of Kalman filters operating in parallel. The Kalman-type correction reduces the risk of ensemble collapse, which enables the filter to efficiently operate with fewer particles than the PF.

In this contribution, we present the PKF and discuss how the ensemble Kalman filtering (EnKF) methods can be derived from the PKF. We argue that the (deterministic) Square-Root EnKFs are Gaussian-based filters while the traditional (stochastic) EnKF propagates an approximation of the non-Gaussian pdf of the state. We also discuss approaches to reduce the computational burden of the PKF to make it suitable for realistic ocean assimilation problems. These are based on low-rank approximations of the Gaussian sum covariance matrices. Results of numerical experiments with the strongly nonlinear Lorenz-96 model will be discussed.